

## ACADEMIC SCHOLARSHIP 2009

## MATHEMATICS

PAPER 2
Two Hours

## CALCULATORS WILL BE NEEDED FOR THIS PAPER

## INSTRUCTIONS TO CANDIDATES

You are not expected to have time to do all the questions.
You may answer the questions in any order.
Choose those questions which you think you can answer best.
Remember to show your working and clearly show the method you are using.
Take $\pi$ as either 3.14 or the value given on your calculator.
Answers should be given to 3 significant figures where appropriate.
Some questions are longer than others.
The number of marks for each question is shown in square brackets.


1. On Monday, Sian sent 16 messages on the Twitter website. On Tuesday she sent 41.
a) What percentage increase does this represent?

On Thursday she sent 99 Twitter messages, a 32\% increase on Wednesday's total.
b) How many did she send on Wednesday?
2. Find all solutions to:
a) $x^{2}-16=2009$
b) $\left(x^{2}-17\right)^{2}+36=100$
3. Let $a=3.84, b=-7.31, c=-20.39$.

Writing down all digits on your calculator display, calculate the value of:
a) $2 a-3 c$
b) $a^{2}-b^{2}$
c) $\frac{a}{3 b}$
d) $\sqrt{b-c}$
4. Triangle ABC is isosceles, with $\mathrm{AB}=\mathrm{BC}$.

As indicated in the diagram, $\mathrm{AC}=\mathrm{AD}=\mathrm{BD}$.
By letting angle $\mathrm{DAB}=x$, or otherwise, find the size of angle DAC.

[9]
5. At 11:00am, a car leaves the city of Burne at a constant speed of 60 mph towards the city of Newleigh. At the same time a second car leaves Newleigh and drives toward Burne at the constant speed of 50 mph . The distance between the two cities is 220 miles and they are connected by a long straight road.
a) What is the combined distance travelled by both cars at the point of meeting?
b) At what time will the two cars meet each other?
c) If the two drivers agree to leave at 11 am and to meet instead at $3 \mathrm{pm}, 20$ miles closer to Burne, what would their constant speeds need to be?
6. Gemma rolls a regular six-sided die, numbered 1 to 6 . Sophie rolls a regular eight-sided die, numbered 1 to 8 . Find the probability that:
a) both girls roll a 6 .
b) Gemma scores more than Sophie.
c) at least one of the two girls scores a prime number.
d) the difference between their scores is less than 3 .
7. Steve ("The Shark") is playing pool on a rectangular table ABCD measuring 6 feet by 3 feet. He fires the ball from the middle of the cushion AD, so that it hits the cushion AB , and rebounds into pocket C .

Given that angle $\mathrm{ANM}=$ angle BNC ,
a) Find distance AN

b) Calculate the distance that the ball travels
c) Calculate the area of triangle MNC.
(In this question, units will be in feet, or feet ${ }^{2}$ )
8. A rectangle is made up of six squares, as shown in the diagram (not to scale). The smallest square has area $1 \mathrm{~cm}^{2}$.

By letting the square in the top right corner have side $x$, or otherwise, find the area of the whole rectangle.

[10]
9. Anthony and Brian run clockwise around a rectangular field of length 200 m and width 150 m , at a constant speed of 5 metres per second. They start at opposite corners, as shown in the diagram.

Draw a sketch graph to show how the distance between them, measured in a direct straight
 line, changes during one lap of the track. You should put time on the horizontal axis, and distance on the vertical axis.
(Credit will be given for marking key points on your graph, and for getting the shape correct.)
10. Clarissa makes a $2 \times 2 \times 2$ cube out of straws of length 1 , as in the diagram. At each point where two straws meet, a plastic connector is used.
a) How many of these connectors join:
i.) 3 straws
ii.) 4 straws

iii.) 5 straws
iv.) 6 straws
b) She then extends the structure to make a $20 \times 20 \times 20$ cube (she has a lot of patience, and straws!). How many plastic connectors now join:
i.) 3 straws
ii.) 4 straws
iii.) 5 straws
iv.) 6 straws
11. An isosceles triangle has sides $5 \mathrm{~cm}, 5 \mathrm{~cm}$ and 6 cm .
a) Find its area
b) Calculate the perpendicular height $h$.

The triangle is cut into two triangles with different areas but equal perimeters.

c) Find the areas of these two pieces.
12. Rectangle ABCD is divided into three parts by two lines, EC and FD.
$F$ is the midpoint of the line $E C$, and point $E$ is on $A B$ so that the ratio $\mathrm{AE}: \mathrm{EB}$ is $1: k$.

a) Find the area of triangle CDF.
b) Find the area of triangle EBC in terms of $k$.
c) Find the area of quadrilateral AEFD in terms of $k$.
d) For what value of $k$ does the quadrilateral AEFD cover exactly a third of the area of rectangle ABCD ?

